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Interpolating fields for spin-1 mesons

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Three commonly used types of effective theories for vector mesons are shown to correspond to different choices of interpolating field for spin-1 particles and the rules for transforming between them are described. The choice of fields that transform homogeneously under the nonlinear realisation of chiral symmetry imposes no preconceptions about the types of coupling for the mesons, and so this representation is particularly useful for comparing different theories. By converting them into this form, hidden-gauge theories are shown to contain an adjustable parameter, the gauge coupling. The normal choice for this is seen to be one that reduces the momentum dependence of the effective $\rho\pi\pi$ coupling, explaining the success of the "low-energy theorem" of that approach.

I. INTRODUCTION

At very low energies strong interactions among pions can be described by an effective Lagrangian based on a chirally symmetric sigma model [1]. To extend such a description to higher energies heavier mesons need to be incorporated, most notably vector mesons. Various schemes for doing so have been proposed, differing in the transformation properties of their vector fields under chiral symmetry.

Many of these approaches are motivated by the phenomenologically successful ideas of vector-meson dominance and universal coupling [2]. These lead to kinetic terms and

couplings for the spin-1 mesons that have the same forms as in a gauge theory, reflecting the assumed universal coupling of these mesons to conserved currents. Examples include the "massive Yang-Mills" [3–5] and "hidden-gauge" theories [6]. However it is not necessary to impose such a structure on the effective Lagrangian from the start. An alternative scheme for incorporating these mesons was suggested by Weinberg [7] and developed further by Callan, Coleman, Wess and Zumino [8]. In this treatment, denoted here by WCCWZ, the fields transform homogeneously under a nonlinear realisation of chiral symmetry. Another, related approach is that of Ecker *et al.* in which the spin-1 mesons are represented by antisymmetric tensor fields [9,10].

Despite the rather different forms of their Lagrangians, and the different types of coupling contained in them, all of these approaches are in principle equivalent. Each corresponds to a different choice of interpolating fields for the spin-1 mesons. This is illustrated rather well in extended Nambu–Jona-Lasinio models [11,12], where there is considerable freedom in the choice of auxiliary fields in the vector and axial channels. To some extent the choice of scheme must be based on the simplicity of the resulting Lagrangian. In making comparisons between the approaches it is important not to confuse features that arise from the choice of interpolating field with those that arise from requiring, for instance, universal coupling of the vector mesons. The former are not physical, controlling merely the off-shell behaviour of scattering amplitudes. The latter do have physical consequences, such as relations between on-shell amplitudes for different processes.

Interest in these Lagrangians has recently been reawakened by the possibility that experiments at high-luminosity accelerators such as CEBAF or DAΦNE may be able to explore some of the couplings that have up to now been inaccessible. Measurements of these may be able to discriminate between the proposed effective theories [13,14]. These theories are also currently being used to predict the behaviour of vector mesons in hot and dense matter [15,16] and some of them lead to quite different predictions for the mass of the ρ in matter [17]. In both of these contexts it is important to be able to compare theories, which may be expressed in different formalisms, in a way which is independent of the different choices of

fields. To this end I explore here the connections between the WCCWZ, hidden-gauge and massive Yang-Mills approaches, and their corresponding interpolating fields.

The WCCWZ scheme, described in Sec. II, is particularly useful for comparisons between theories since it imposes no prejudices about the forms of the couplings among the mesons. As noted by Ecker *et al.* [10], the consequences of physical assumptions like vector dominance can then be rather transparently expressed as relations between the couplings in a WCCWZ Lagrangian. By converting commonly used hidden-gauge and massive Yang-Mills theories into their WCCWZ equivalents, their couplings can be directly compared.

In the hidden-gauge approach an artificial local symmetry is introduced into the nonlinear sigma model by the choice of field variables. The ρ meson is then introduced as a gauge boson for this symmetry. As stressed by Georgi [18], the additional local symmetry has no physics associated with it, and it can be removed by fixing the gauge. In the unitary gauge the symmetry reduces to a nonlinear realisation of chiral symmetry, under which the vector fields transform inhomogeneously, in contrast to those of WCCWZ. However, with a further change of variable any vector-meson Lagrangian of the hidden-gauge form can be converted into an equivalent WCCWZ one [18]. The rules for transforming a Lagrangian from hidden-gauge to WCCWZ form have also been noted by Ecker *et al.* [10]. In Sec. III, this equivalence is shown to hold for general hidden-gauge theories with axial as well as vector mesons.

As I show here, by changing variables from the hidden-gauge to WCCWZ scheme, the gauge coupling constant of the former is really a parameter in the choice of interpolating vector field. This coupling does not appear in the equivalent WCCWZ Lagrangian and so different hidden-gauge theories with different gauge couplings can be equivalent. The conventional choice is shown to be one that eliminates any $\mathcal{O}(p^3)$ $\rho\pi\pi$ coupling from the hidden-gauge Lagrangian, so that the leading corrections to the $\mathcal{O}(p)$ coupling are of order $\mathcal{O}(p^5)$. This reduction of the momentum dependence of the coupling can explain why the "low-energy theorem" of the hidden-gauge approach [19,20] is well satisfied by the $\rho\pi\pi$ coupling determined from the decay of on-shell ρ mesons.

In an appendix I describe the relation between the WCCWZ and massive Yang-Mills theories [3,5]. In the latter, the vector and axial fields transform under a linear realisation of chiral symmetry. Three- and four-point couplings among these fields are included and, together with the kinetic terms, form a Yang-Mills Lagrangian with a local chiral symmetry. The full theory does not possess this gauge symmetry since it includes mass terms which have only global symmetry. By changing variables to spin-1 fields that transform under the nonlinear realisation of chiral symmetry, any massive Yang-Mills theory can be converted into an equivalent WCCWZ one.

Assumptions about the couplings implicit in commonly used hidden-gauge and massive Yang-Mills theories can be made manifest by converting both to WCCWZ form. The close relationship between these theories can also be seen when they are expressed in this form. In particular both give rise to four-point couplings that arise from assuming resonance saturation in the corresponding scattering processes [9,10]. In the simplest hidden-gauge Lagrangian for pions and ρ mesons [21], the $\rho\pi\pi$ and 3ρ couplings are shown to be related by the assumed universal coupling in these models.

II. WCCWZ

The WCCWZ scheme is based on the nonlinear realisation of chiral symmetry introduced by Weinberg [7]. The starting point for this is a two-flavour nonlinear sigma model, defined in terms of the unitary matrix constructed out of the pion fields, $U(x) = \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)/f_\pi)$. Under the global SU(2)×SU(2) chiral symmetry this transforms as

$$U(x) \to g_L U(x) g_R^\dagger, \tag{2.1}$$

with $g_L, g_R \in SU(2)$. The nonlinear realisation of the symmetry is obtained from the transformation properties of the square root of U, denoted by u:

$$u(x) \to g_L u(x) h^\dagger \left(u(x), g_L, g_R\right) = h \left(u(x), g_L, g_R\right) u(x) g_R^\dagger, \tag{2.2}$$

where $h\left(u(x), g_L, g_R\right)$ is a compensating SU(2) rotation which depends on the pion fields at x as well as $g_{L,R}$. The detailed form of h is not needed here; it can be found in Ref. [8]. It is convenient to introduce the following field gradients

$$u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$$
$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \tag{2.3}$$

which behave as an axial vector and vector respectively. Under chiral rotations these transform as

$$u_\mu \rightarrow h u_\mu h^\dagger$$
$$\Gamma_\mu \rightarrow h \Gamma_\mu h^\dagger + h \partial_\mu h^\dagger. \tag{2.4}$$

The quantity u_μ is seen to transform homogeneously whereas the transformation of Γ_μ is inhomogeneous. In fact Γ_μ is the connection on the coset space SU(2)×SU(2)/SU(2) and it can be used to construct the covariant derivative on this space:

$$\nabla_\mu = \partial_\mu + [\Gamma_\mu, \ ]. \tag{2.5}$$

The covariant derivatives of u_μ satisfy the useful relation

$$\nabla_\mu u_\nu - \nabla_\nu u_\mu = 0. \tag{2.6}$$

Also, the curvature tensor corresponding to Γ_μ can be expressed in terms of u_μ as

$$\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] = \frac{1}{4}[u_\mu, u_\nu], \tag{2.7}$$

In the WCCWZ approach [7,8,10] vector and axial fields transform homogeneously under this symmetry

$$V_\mu \rightarrow h V_\mu h^\dagger$$
$$A_\mu \rightarrow h A_\mu h^\dagger, \tag{2.8}$$

where $V_\mu = \frac{1}{2}\boldsymbol{\tau} \cdot \mathbf{V}_\mu$ and $A_\mu = \frac{1}{2}\boldsymbol{\tau} \cdot \mathbf{A}_\mu$. It is convenient to define covariant derivatives of these fields,

$$V_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu, \qquad A_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \tag{2.9}$$

A general chirally symmetric Lagrangian for $\pi\rho a_1$ physics consists of all terms that can be constructed out of traces of products of u_μ, V_μ, A_μ, $V_{\mu\nu}$, $A_{\mu\nu}$ and their covariant derivatives, and that are symmetric under parity. For example, up to fourth-order in pion-field gradients and the vector fields, the Lagrangian includes the terms

$$\mathcal{L} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu\rangle - \frac{1}{2}\langle V_{\mu\nu}V^{\mu\nu}\rangle + m_V^2\langle V_\mu V^\mu\rangle - \frac{i}{2}g_1\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \frac{i}{2}g_2\langle V_{\mu\nu}[V^\mu, V^\nu]\rangle$$
$$+\frac{1}{8}g_3\langle [u_\mu, u_\nu]^2\rangle - \frac{1}{4}g_4\langle [u_\mu, u_\nu][V^\mu, V^\nu]\rangle + \frac{1}{8}g_5\langle [V_\mu, V_\nu]^2\rangle + \cdots, \qquad (2.10)$$

where $\langle\cdots\rangle$ denotes a trace in SU(2) space. The terms written out explicitly here are the ones we shall need in discussing the connection to the hidden-gauge Lagrangian of Bando *et al.* [21]. These include the famous $\langle [u_\mu, u_\nu]^2\rangle$ term introduced by Skyrme to stabilise solitons in a nonlinear sigma model [22]. Obviously many other three- and four-point interactions, involving the axial as well as the vector field, should also be present in the full effective Lagrangian.

As pointed out by Kalafatis [23] and discussed further elsewhere [24], the four-point couplings must satisfy inequalities relating them to the three-point couplings if the Hamiltonian corresponding to (2.10) is to be bounded from below. For example the coefficient of the Skyrme term should satisfy $g_3 \geq g_1^2$. If one assumes that vector dominance holds in the strong interaction and that the scattering processes corresponding to these four-point couplings are saturated by exchange of a single resonance, in this case the ρ, then the equalities hold, for example $g_3 = g_1^2$, as discussed in [10].

III. HIDDEN-GAUGE THEORIES

In the simplest version of the hidden-gauge approach, the gauge symmetry is just SU(2) and only vector mesons are treated as gauge bosons [21]. The extension to axial-vector mesons requires a local SU(2)×SU(2) symmetry, which can be introduced by writing $U(x)$ as a product of three unitary matrices [19,5,6],

$$U(x) = \xi_L(x)^\dagger \xi_M(x)\xi_R(x). \qquad (3.1)$$

Since at each point in space-time this factorisation is arbitrary, the new variables are symmetric under

$$\xi_R(x) \to h_R(x)\xi_R(x)g_R^\dagger$$

$$\xi_L(x) \to h_L(x)\xi_L(x)g_L^\dagger$$

$$\xi_M(x) \to h_L(x)\xi_M(x)h_R^\dagger(x), \tag{3.2}$$

where $h_{L,R}(x)$ are SU(2) matrices with arbitrary x-dependence. The freedom to make space-time dependent rotations of $\xi_{r,l,m}(x)$ in this way provides the local SU(2)×SU(2) symmetry of this scheme.

One can always to choose to work in the unitary gauge where

$$\xi_R(x) = \xi_L^\dagger(x) = u(x), \qquad \xi_M(x) = 1, \tag{3.3}$$

for all x. The symmetry (3.1) then reduces to the usual nonlinear realisation of chiral symmetry [7,8], as in Eq. (2.2), where the x dependence of $h_R(x) = h_L(x)$ is no longer arbitrary but is given in terms of the pion fields. This gauge fixing thus provides the basis for translating between the hidden-gauge and WCCWZ formalisms.

In this approach, spin-1 fields are introduced as gauge bosons of this artificial local symmetry. Right- and left-handed gauge fields transform under the symmetry as, respectively,

$$\widehat{X}_\mu(x) \to h_R(x)\widehat{X}_\mu(x)h_R^\dagger(x) + \frac{i}{\sqrt{2}g}h_R(x)\partial_\mu(x)h_R^\dagger(x)$$

$$\widehat{Y}_\mu(x) \to h_L(x)\widehat{Y}_\mu(x)h_L^\dagger(x) + \frac{i}{\sqrt{2}g}h_L(x)\partial_\mu(x)h_L^\dagger(x), \tag{3.4}$$

where I use hats to distinguish the hidden-gauge spin-1 fields from those of the WCCWZ approach. The corresponding gauge-covariant field strengths are

$$\widehat{X}_{\mu\nu} = \partial_\mu\widehat{X}_\nu - \partial_\nu\widehat{X}_\mu - i\sqrt{2}g[\widehat{X}_\mu, \widehat{X}_\nu]$$

$$\widehat{Y}_{\mu\nu} = \partial_\mu\widehat{Y}_\nu - \partial_\nu\widehat{Y}_\mu - i\sqrt{2}g[\widehat{Y}_\mu, \widehat{Y}_\nu]. \tag{3.5}$$

It is usually more convenient to work in terms of the vector and axial fields, $\widehat{V}_\mu = (\widehat{X}_\mu + \widehat{Y}_\mu)/\sqrt{2}$ and $\widehat{A}_\mu = (\widehat{X}_\mu - \widehat{Y}_\mu)/\sqrt{2}$. The field strengths for these are

$$\widehat{V}_{\mu\nu} = \partial_\mu\widehat{V}_\nu - \partial_\nu\widehat{V}_\mu - ig[\widehat{V}_\mu, \widehat{V}_\nu] - ig[\widehat{A}_\mu, \widehat{A}_\nu]$$

$$\widehat{A}_{\mu\nu} = \partial_\mu \widehat{A}_\nu - \partial_\nu \widehat{A}_\mu - ig[\widehat{V}_\mu, \widehat{A}_\nu] - ig[\widehat{A}_\mu, \widehat{V}_\nu]. \tag{3.6}$$

The gauge-covariant first derivatives of the pion fields are

$$\begin{aligned} R_\mu &= -i\left[(\partial_\mu \xi_L)\xi_L^\dagger - i\sqrt{2}g\widehat{X}_\mu\right] \\ L_\mu &= -i\left[(\partial_\mu \xi_R)\xi_R^\dagger - i\sqrt{2}g\widehat{Y}_\mu\right] \\ M_\mu &= -i\left[(\partial_\mu \xi_M)\xi_M^\dagger + i\sqrt{2}g\xi_M\widehat{X}_\mu\xi_M^\dagger - i\sqrt{2}g\widehat{Y}_\mu\right]. \end{aligned} \tag{3.7}$$

Of these, R_μ transforms covariantly under the right-handed local symmetry, L_μ and M_μ under the left-handed. A general gauge-invariant Lagrangian in this approach consists of all terms that can be constructed out of traces of products of R_μ, L_μ, M_μ, $\widehat{X}_{\mu\nu}$, $\widehat{Y}_{\mu\nu}$, and their covariant derivatives, and that are symmetric under parity. Factors of ξ_M and $\xi_M^\dagger$ should be inserted between right- and left-covariant quantities. Writing out explicitly only terms of second order in the pion field gradients (which also provide mass terms for the heavy mesons) and the vector-meson kinetic terms, one has the Lagrangian [19,5,6]

$$\begin{aligned} \mathcal{L} = &\frac{af_\pi^2}{4}\langle(L_\mu + \xi_M R_\mu \xi_M^\dagger)^2\rangle + \frac{bf_\pi^2}{4}\langle(L_\mu - \xi_M R_\mu \xi_M^\dagger)^2\rangle \\ &+\frac{cf_\pi^2}{4}\langle M_\mu M^\mu\rangle + \frac{df_\pi^2}{4}\langle(L_\mu - \xi_M R_\mu \xi_M^\dagger - M_\mu)^2\rangle \\ &-\frac{1}{2}\langle \widehat{X}_{\mu\nu}\widehat{X}^{\mu\nu} + \widehat{Y}_{\mu\nu}\widehat{Y}^{\mu\nu}\rangle + \cdots. \end{aligned} \tag{3.8}$$

In the unitary gauge defined by (3.3), the transformation properties of the spin-1 fields are

$$\begin{aligned} \widehat{V}_\mu &\to h\widehat{V}_\mu h^\dagger + \frac{i}{g}h\partial_\mu h^\dagger \\ \widehat{A}_\mu &\to h\widehat{A}_\mu h^\dagger, \end{aligned} \tag{3.9}$$

where h is the compensating SU(2) rotation of the nonlinear realisation of chiral symmetry, Eq. (2.2). The covariant gradients of Eq. (3.7) reduce to

$$R_\mu = iu^\dagger \partial_\mu u - g(\widehat{V}_\mu + \widehat{A}_\mu)$$

$$L_\mu = iu\partial_\mu u^\dagger - g(\widehat{V}_\mu - \widehat{A}_\mu)$$

$$M_\mu = 2g\widehat{A}_\mu. \tag{3.10}$$

Since these can be combined to give

$$R_\mu + L_\mu = 2(i\Gamma_\mu - g\widehat{V}_\mu)$$

$$R_\mu - L_\mu = u_\mu - 2g\widehat{A}_\mu, \tag{3.11}$$

we see that $\widehat{V}_\mu$ always appears in the combination

$$V_\mu = \widehat{V}_\mu - \frac{i}{g}\Gamma_\mu. \tag{3.12}$$

This transforms homogeneously under the nonlinear chiral rotation, as can be seen from (2.4) and (3.9).

In the unitary gauge we can therefore change variables to the vector field V_μ of Eq. (3.12) to obtain a Lagrangian of the WCCWZ type. (The axial field already transforms homogeneously in this gauge, Eq. (3.9).) With the aid of Eq. (2.7), the field strengths of Eq. (3.6) can be expressed in terms of the new field as

$$\widehat{V}_{\mu\nu} = V_{\mu\nu} + \frac{i}{4g}[u_\mu, u_\nu] - ig[V_\mu, V_\nu] - ig[A_\mu, A_\nu]$$

$$\widehat{A}_{\mu\nu} = A_{\mu\nu} - ig[V_\mu, A_\nu] - ig[A_\mu, V_\nu], \tag{3.13}$$

where the covariant field gradients are defined in (2.9) above. Terms involving higher gauge-covariant derivatives can be rewritten in terms of the covariant derivative (2.5) using

$$D_\mu = \partial_\mu - ig[\widehat{V}_\mu,\ ] = \nabla_\mu - ig[V_\mu,\ ]. \tag{3.14}$$

Each term of the general hidden-gauge Lagrangian in the unitary gauge has a corresponding term in the general WCCWZ Lagrangian, where $\widehat{V}_\mu - i\Gamma_\mu/g$ has been replaced by V_μ, D_μ by ∇_μ, $\widehat{V}_{\mu\nu}$ by $V_{\mu\nu}$, and $\widehat{A}_{\mu\nu}$ by $A_{\mu\nu}$. The coupling constants will not be identical but, if one takes account of Eqs. (3.13, 14), there is a well defined way to convert from one approach to the other. This generalises Georgi's observation [18] of the equivalence of the two formalisms to the case of axial as well as vector fields.

An important feature to note is that the gauge coupling constant g of the hidden-gauge approach does not appear in the WCCWZ approach. Indeed different hidden-gauge Lagrangians with different values of g can be equivalent to the same WCCWZ theory. This should not be too surprising: the local symmetry is not physical but arises from a particular choice of field variables in Eq. (3.1) and hence the corresponding coupling is not a physical quantity. The significance of g becomes clearer if one starts from a WCCWZ Lagrangian and converts it into a hidden-gauge one using Eq. (3.12) in reverse. Any value of g can be used in (3.12) to define a new vector field $\widehat{V}_\mu$ and the resulting Lagrangian will have the form of a hidden-gauge theory in the unitary gauge. Different choices of g therefore correspond to different choices of interpolating vector field. The value of g should thus be fixed by considerations of calculational convenience, for example the elimination of certain types of term from the effective Lagrangian.

To explore this equivalence in more detail, let us examine a specific hidden-gauge theory. The example considered is the most commonly used hidden-gauge model, introduced by Bando *et al.* [21]. This contains a vector but no axial field and so is invariant under the diagonal SU(2) subgroup of the local symmetry only. Its Lagrangian has the form

$$\mathcal{L} = \frac{f_\pi^2}{4}\langle (L_\mu - R_\mu)^2\rangle + \frac{a f_\pi^2}{4}\langle (L_\mu + R_\mu)^2\rangle - \frac{1}{2}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu}\rangle, \tag{3.15}$$

where

$$\begin{aligned} R_\mu &= -i\left[(\partial_\mu \xi_L)\xi_L^\dagger - ig\widehat{V}_\mu\right] \\ L_\mu &= -i\left[(\partial_\mu \xi_R)\xi_R^\dagger - ig\widehat{V}_\mu\right]. \end{aligned} \tag{3.16}$$

In the unitary gauge this becomes

$$\mathcal{L} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu\rangle + a f_\pi^2\langle (i\Gamma_\mu - g\widehat{V}_\mu)^2\rangle - \frac{1}{2}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu}\rangle, \tag{3.17}$$

showing that the ρ mass is given in terms of the parameter a by

$$m_V^2 = a g^2 f_\pi^2. \tag{3.18}$$

Using (3.12, 13) this can be expressed in the form of the WCCWZ Lagrangian of Eq. (2.10), with the following values for the coupling constants:

$$g_1 = \frac{1}{2g}, \quad g_2 = 2g, \quad g_3 = \frac{1}{4g^2}, \quad g_4 = 1, \quad g_5 = 4g^2. \tag{3.19}$$

The couplings in this model thus satisfy the relations

$$g_3 = g_1^2, \quad g_4 = g_1 g_2, \quad g_5 = g_2^2, \tag{3.20}$$

which arise from assuming vector meson dominance in the strong interaction and saturation of the four-point couplings by the ρ-meson alone, as discussed following Eq. (2.10). The other condition that defines the model is a relation between the $\rho\pi\pi$ and 3ρ couplings,

$$g_1 = \frac{1}{g_2}. \tag{3.21}$$

These relations (3.20, 21) allow the three- and four-point couplings to be combined into a kinetic term for the vector field with a Yang-Mills form. Note that all of these relations hold for any value of the ρ mass (or equivalently of the parameter a).

Complete vector dominance of the pion electromagnetic coupling is obtained in this model if the ρ mass satisfies the KSRF relation [25] (in its second form)

$$m_V^2 = 2g^2 f_\pi^2, \tag{3.22}$$

In terms of the parameters of Bando *et al.* [21,6] this corresponds to $a = 2$. Vector dominance in the couplings of the photon to all hadrons requires universal coupling of the ρ meson to the conserved isospin current. By examining the $\rho\pi\pi$ coupling contained in (3.17),

$$-2igaf_\pi^2 \langle \widehat{V}^\mu \Gamma_\mu \rangle = \frac{1}{2} ag \widehat{\mathbf{V}}^\mu \cdot \boldsymbol{\pi} \wedge \partial_\mu \boldsymbol{\pi} + \mathcal{O}(\pi^4), \tag{3.23}$$

and the 3ρ coupling

$$2ig\langle (\partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu)[\widehat{V}^\mu, \widehat{V}^\nu] \rangle = -g\widehat{\mathbf{V}}^\mu \cdot \widehat{\mathbf{V}}^\nu \wedge \partial_\mu \widehat{\mathbf{V}}_\nu. \tag{3.24}$$

we see that for $a = 2$ the couplings have the same strength and so the model embodies universal coupling of the ρ to itself and to the pion.

In looking at predictions of the model (3.15), consequences of the hidden-gauge choice of interpolating field should not be confused with those arising from the relations between the coupling constants (3.20, 21). The former controls merely the form of off-shell extrapolations of those amplitudes. The latter lead to relations between amplitudes for physical processes, and are of course specific to the choice of Lagrangian. For example the relation (3.21) between the $\rho\pi\pi$ and 3ρ couplings can be removed without violating the hidden-gauge invariance by adding a term of the form $\langle \widehat{V}_{\mu\nu}\,[R^\mu + L^\mu, R^\nu + L^\nu]\rangle$ to the Lagrangian (3.15). This is invariant under the same local SU(2) symmetry as the rest of the Lagrangian. It provides an additional contribution to the 3ρ coupling beyond that in the kinetic term. After gauge-fixing and change of variables, such a term would lead to an equivalent WCCWZ Lagrangian that would not satisfy (3.21).

To see why the specific choice of field in (3.15) is particularly convenient, consider the $\rho\pi\pi$ of the corresponding WCCWZ Lagrangian (2.10),

$$-\frac{i}{2}g_1\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle = g_1\partial^\mu \mathbf{V}^\nu \cdot \partial_\mu \boldsymbol{\pi} \wedge \partial_\nu \boldsymbol{\pi}, \tag{3.25}$$

This is of third-order in the momenta. In contrast, when the model is expressed in hidden-gauge form (3.15), the leading $\rho\pi\pi$ coupling (3.23) is of first order in the momenta of the particles involved. Using the interpolating field defined by (3.12) with the constant g given by

$$g = \frac{1}{2g_1} \tag{3.26}$$

eliminates any $\mathcal{O}(p^3)$ term from the hidden-gauge Lagrangian. This happens even if the initial WCCWZ Lagrangian contains other, higher-derivative $\rho\pi\pi$ couplings. The advantage of the hidden-gauge choice of interpolating field with this g is that any corrections to the leading $\rho\pi\pi$ of Eq. (3.23) are at least of order $\mathcal{O}(p^5)$. Provided m_V is small compared with the scale at which physics beyond the $\pi\rho$ Lagrangian becomes significant, the momentum dependence of the effective $\rho\pi\pi$ coupling should be small in the hidden-gauge representation. This explains why the KSRF relation [25] in its first version, which relates the $\rho\pi\pi$ coupling and $\gamma\rho$ mixing at zero four-momentum and forms the "low-energy theorem" of the hidden-

gauge approach [19,20], is actually rather well satisfied by the values for on-shell ρ mesons. The freedom to choose g in this way after renormalisation can also explain why the low-energy theorem continues to hold when loop corrections are included [26,27].

IV. CONCLUSIONS

As described here any effective theories of spin-1 mesons and pions can be expressed in either WCCWZ, hidden-gauge or massive Yang-Mills form. These formalisms correspond to different choices of interpolating fields for the spin-1 mesons. The rules for transforming a theory from one form to another generalise the equivalences between the approaches that have previously been noted for particular cases. Since all the formalisms are equivalent, the choice between them must depend on the convenience of the corresponding Lagrangians for a specific calculation.

The hidden-gauge case is of particular interest since it involves a vector field that depends on a continuous parameter g, which acts as the gauge coupling for the local symmetry introduced in this approach. This parameter can be chosen to remove the $\mathcal{O}(p^3)$ momentum dependence of the $\rho\pi\pi$ coupling, and indeed this choice is implicitly used in applications of the hidden-gauge approach. This reduction of the momentum dependence of the $\rho\pi\pi$ coupling can explain the success of the "low-energy theorem" of this approach. It suggests that this may be a particularly convenient representation to use for low-energy $\pi\rho$ physics.

The massive Yang-Mills representation (discussed in the Appendix) leads to no such simplifications. It is however based on spin-1 fields that transform linearly under chiral rotations. This may be of use in studying the restoration of chiral symmetry if Weinberg's ideas of "mended" chiral symmetry [28] are relevant in this context, as has been suggested by Brown and Rho [16].

The WCCWZ formalism is particularly useful as framework for comparing different theories and for elucidating the implicit relations between their couplings. It shows that commonly used hidden-gauge and massive Yang-Mills theories both give rise to four-point

couplings that arise from assuming resonance saturation in the corresponding scattering processes [9,10]. In the hidden gauge case the $\rho\pi\pi$ and 3ρ couplings are related by the assumption of universal coupling. Although this is not the case in the simplest massive Yang-Mills theory, non-minimal terms can be added to such a theory so that it leads to similar predictions. Such assumptions about the couplings in these models could be tested experimentally using processes that are sensitive to the 3ρ coupling, for example $\rho \to \pi^+\pi^-2\pi^0$ [14].

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APPENDIX: MASSIVE YANG-MILLS

The massive Yang-Mills approach [3–5] is based on vector and axial fields that transform linearly under the SU(2)×SU(2) symmetry. The right- and left-handed combinations of these spin-1 fields, denoted here by $\widetilde{X}_\mu$ and $\widetilde{Y}_\mu$, transform as

$$\begin{aligned} \widetilde{X}_\mu &\to g_R \widetilde{X}_\mu g_R^\dagger \\ \widetilde{Y}_\mu &\to g_L \widetilde{Y}_\mu g_L^\dagger. \end{aligned} \tag{A.1}$$

The Lagrangian for these is chosen to contain kinetic terms of the Yang-Mills form, including three- and four-point interactions. The couplings of the spin-1 fields to pions are also chosen to have a gauge-invariant form, ensuring universal coupling of the ρ and allowing photons to be coupled in a way consistent with vector dominance. Although the interaction terms respect a local SU(2)×SU(2) symmetry, the full theory does not since it also includes mass terms for the spin-1 mesons.

A simple massive Yang-Mills Lagrangian, which illustrates the features of the approach, consists of the gauged sigma model (a nonlinear version of the model used by Gasiorowicz and Geffen [3])

$$\mathcal{L} = \frac{f_0^2}{4}\langle \widetilde{D}_\mu U(\widetilde{D}_\mu U)^\dagger\rangle - \frac{1}{2}\langle \widetilde{X}_{\mu\nu}\widetilde{X}^{\mu\nu} + \widetilde{Y}_{\mu\nu}\widetilde{Y}^{\mu\nu}\rangle$$
$$+m_V^2\langle \widetilde{X}_\mu\widetilde{X}^\mu + \widetilde{Y}_\mu\widetilde{Y}^\mu\rangle, \tag{A.2}$$

where

$$\widetilde{D}_\mu U = \partial_\mu U + i\sqrt{2}\widetilde{g}U\widetilde{X}_\mu - i\sqrt{2}\widetilde{g}\widetilde{Y}_\mu U, \tag{A.3}$$

and the field strengths $\widetilde{X}_{\mu\nu}$, $\widetilde{Y}_{\mu\nu}$ are defined analogously to those in Eq. (3.5).

The massive Yang-Mills theory can be converted into an equivalent WCCWZ one by using $u(x)$ to construct spin-1 fields that transform under the nonlinear realisation of chiral symmetry (2.8):

$$X_\mu = u\widetilde{X}_\mu u^\dagger$$
$$Y_\mu = u^\dagger\widetilde{Y}_\mu u. \tag{A.4}$$

The kinetic terms can be expressed in terms of the covariant derivatives of these fields, defined as in (2.9), using

$$\widetilde{X}_{\mu\nu} = u^\dagger\left[X_{\mu\nu} + \frac{i}{2}[u_\mu, X_\nu] - \frac{i}{2}[u_\nu, X_\mu] - i\sqrt{2}\widetilde{g}[X_\mu, X_\nu]\right]u$$
$$\widetilde{Y}_{\mu\nu} = u^\dagger\left[Y_{\mu\nu} - \frac{i}{2}[u_\mu, Y_\nu] + \frac{i}{2}[u_\nu, Y_\mu] - i\sqrt{2}\widetilde{g}[Y_\mu, Y_\nu]\right]u. \tag{A.5}$$

In terms of u_μ and these fields, the pion kinetic term can be written

$$\langle \widetilde{D}_\mu U(\widetilde{D}_\mu U)^\dagger\rangle = \langle [u_\mu - \sqrt{2}\widetilde{g}(X_\mu - Y_\mu)]^2\rangle. \tag{A.6}$$

This contains a πa_1 mixing term which can be removed by an appropriate shift in the definition of the axial field [3,10]. It is thus convenient to define WCCWZ vector and axial fields by

$$V_\mu = \frac{1}{\sqrt{2}}(X_\mu + Y_\mu)$$
$$A_\mu = \frac{1}{\sqrt{2}}(X_\mu - Y_\mu) - \frac{\widetilde{g}f_0^2}{2m_A^2}u_\mu. \tag{A.7}$$

The kinetic terms for the spin-1 fields can then be expressed in terms of V_μ and A_μ and their covariant derivatives (2.9), making use of (2.6). The Lagrangian (A.2) then takes the form

$$\begin{aligned}\mathcal{L} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu\rangle + m_V^2\langle V_\mu V^\mu\rangle + m_A^2\langle A_\mu A^\mu\rangle \\ -\frac{1}{2}\Big\langle\Big\{V_{\mu\nu} - i\widetilde{g}[V_\mu, V_\nu] - i\widetilde{g}[A_\mu, A_\nu] \\ +i\tfrac{1}{2}Z^2\Big([u_\mu, A_\nu] - [u_\nu, A_\mu]\Big) \\ +\frac{i}{4\widetilde{g}}(1-Z^4)[u_\mu, u_\nu]\Big\}^2\Big\rangle \\ -\frac{1}{2}\Big\langle\Big\{A_{\mu\nu} - i\widetilde{g}[V_\mu, A_\nu] - i\widetilde{g}[A_\mu, V_\nu] \\ +i\tfrac{1}{2}Z^2\Big([u_\mu, V_\nu] - [u_\nu, V_\mu]\Big)\Big\}^2\Big\rangle.\end{aligned} \qquad (A.8)$$

where

$$Z^2 = 1 - \frac{\widetilde{g}^2 f_0^2}{m_A^2} = 1 - \frac{\widetilde{g}^2 f_\pi^2}{m_V^2}, \qquad (A.9)$$

and the physical pion decay constant is given by [5]

$$f_\pi^2 = f_0^2 Z^2, \qquad (A.10)$$

and the a_1 mass by

$$m_A^2 = m_V^2/Z^2. \qquad (A.11)$$

Although I have demonstrated the equivalence here for only the theory defined by (A.2), it is general. Any massive Yang-Mills Lagrangian can be expressed in WCCWZ form using (A.4, 5, 7). Conversely, any WCCWZ Lagrangian can be converted into an equivalent massive Yang-Mills theory by inverting these changes of variable. Of course the resulting Lagrangian can contain many terms beyond those present in (A.2), including many non-gauge-invariant interactions. Combined with the results of Section III, this reproduces and generalises the well-known equivalence of the hidden-gauge and massive Yang-Mills formalisms [29–31,5].

By comparing the terms in the Lagrangian (A.8) with the corresponding ones in (2.10), we can see that the couplings satisfy the relations (3.20) arising from assuming resonance

saturation of the four-point interactions. This is similar to the hidden-gauge theory defined by (3.15). The two theories are thus closely related although obviously not identical: the massive Yang-Mills one contains an axial as well as a vector field, and its $\rho\pi\pi$ and 3ρ couplings do not satisfy (3.21). The latter is a consequence of the the additional momentum-dependent $\rho\pi\pi$ couplings that appear after diagonalising in the πa_1 sector. One can always cancel out this momentum dependence by adding extra "nonminimal" terms to the massive Yang-Mills Lagrangian [4].[1] The resulting massive Yang-Mills Lagrangian is then exactly equivalent to the hidden-gauge one involving axial as well as vector mesons introduced in Ref. [20].

Finally, for completeness, I should mention the approach suggested by Brihaye, Pak and Rossi [33] and investigated further by Kuraev, Silagadze and coworkers [34,14]. This is based on a Yang-Mills-type coupling of the ρ, as in the Lagrangian (A.2), but without a chiral partner a_1-field. Simply omitting the axial field from that Lagrangian leaves a theory that is not chirally symmetric. However as described in Ref. [33] additional counterterms can be added to that Lagrangian to ensure that low-energy theorems arising from chiral symmetry are maintained.

Such a theory can be generated by taking a hidden-gauge Lagrangian, such as that of Eq. (3.17), and reversing the procedure above for converting fields that transform linearly under chiral symmetry into ones that transform nonlinearly. Specifically one can define a new vector field $\widetilde{V}_\mu$, related to the hidden-gauge field $\widehat{V}_\mu$ by

$$\widehat{V}_\mu = \frac{1}{2}\left(u^\dagger \widetilde{V}_\mu u + u \widetilde{V}_\mu u^\dagger\right). \tag{A.12}$$

Note that this $\widetilde{V}_\mu$ has no axial partner and so does not transform in any simple way under chiral transformations. By adding and subtracting suitable terms, similar to those in Eq. (A.6),

[1] One can also force the theory into exact equivalence with the one of Bando et al. [21] by imposing a suitable constraint on the axial field [32,5]. This can be seen using the WCCWZ form (A.8): if one demands that $A_\mu = (Z^2/2g)u_\mu$ then one is left with the WCCWZ equivalent of (3.16).

the pion kinetic term can be converted to a form involving gauge-covariant derivatives. The full Lagrangian is then

$$\mathcal{L} = \frac{f_\pi^2}{4}\langle \widetilde{D}_\mu U(\widetilde{D}_\mu U)^\dagger\rangle + \frac{\widetilde{g} f_\pi^2}{2}\langle \widetilde{V}_\mu (u u_\mu u^\dagger - u^\dagger u_\mu u)\rangle - \frac{\widetilde{g}^2 f_\pi^2}{4}\langle (u^\dagger V_\mu u - u V_\mu u^\dagger)^2\rangle$$
$$+ a f_\pi^2 \langle (i\Gamma_\mu - g\widehat{V}_\mu)^2\rangle - \frac{1}{2}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu}\rangle. \qquad (A.13)$$

By choosing the coupling $\widetilde{g}$ to be related to the parameters of the hidden-gauge Lagrangian by

$$\widetilde{g} = \frac{ag}{2}, \qquad (A.14)$$

one can ensure that the $\mathcal{O}(p)$ $\rho\pi\pi$ coupling (3.23) in the fourth term of (A.13) is cancelled by that in the second term. The $\rho\pi\pi$ coupling is then given entirely by the gauge-covariant derivatives in the first term of (A.13).

The resulting theory has a Yang-Mills structure for the ρ kinetic energy and $\rho\pi\pi$ couplings, together with a ρ mass term and a number of additional couplings. These extra couplings are required if the low-energy theorems of chiral symmetry are to be satisfied. They include the counterterms discussed in Refs. [33,34] together with many others. For example the third term of (A.13) contains a momentum-independent $\rho\rho\pi\pi$ coupling. This term is omitted in the calculations of Kuraev *et al.* [34,14] but it is needed to cancel out a corresponding piece of the pion kinetic term, which would otherwise give a nonzero amplitude for $\pi\rho$ scattering at threshold in the chiral limit.

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